DECISION MATHEMATICS (C) UNIT 1 — TEST PAPER 3

1. Use the Shuttle Sort algorithm to arrange the following list of towns in alphabetical order. Show the result of each pass.

   BATH RUGBY LEEDS WIGAN HULL DOVER

   [5]

2. Two algorithms for sorting, A and B, require the following numbers of operations to sort n items:

   Method A: $100n^2 - 3n + 6$ Method B: 0.1×2^n

   Evaluate each one for $n = 10$ and $n = 100$, and comment on their relative performance. [5]

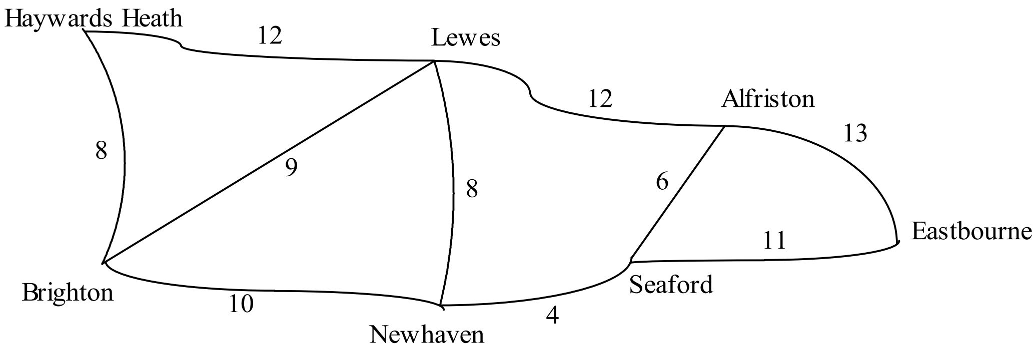
3. First class postage costs 27p and second class costs 19p. A secretary has x letters to send by second class, y letters by first class and z packages (for which postage costs 38p).

   (i) Given a budget of £20 for postage, write down an inequality for x, y and z. [1]

   Generally, there are at least 20 first class letters to post.

   (ii) Explain how the use of the variable $X = x + 2z$ enables the problem to be analysed as a graphical linear programming problem. Find the maximum value of X, and hence find the maximum number of parcels that may be posted. [4]

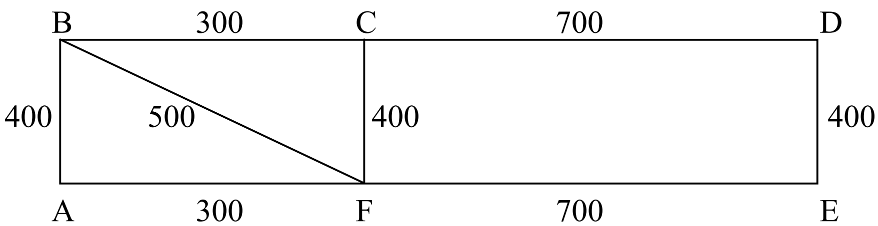
4. A cable TV company wishes to connect the seven towns shown in the following map :



   (i) Use Kruskal's algorithm to find the minimum spanning tree for this network. [6]

   (ii) The area between Seaford and Eastbourne is designated a National Park, and it would cost twice as much, per mile, to lay a cable there as in the rest of the network. Find the tree that would now be the cheapest way of connecting the towns. [2]

5. A doctor has to visit a number of patients on an estate, as shown below.

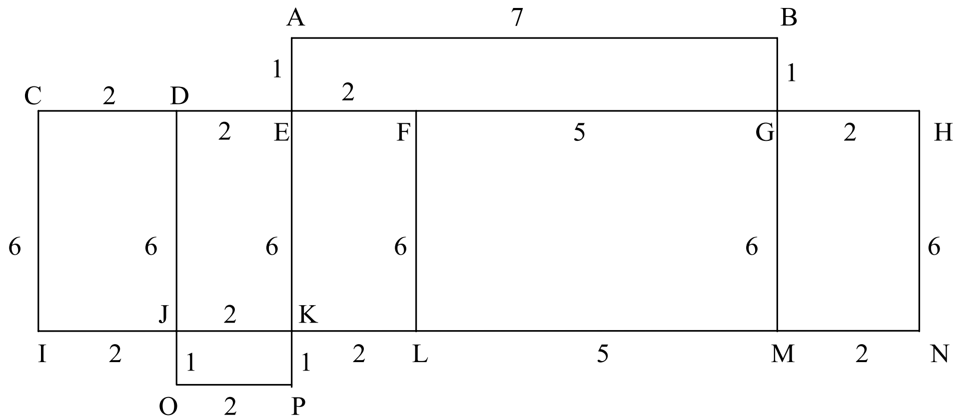


Distances in m.

   (i) By deleting C, find a lower bound for the length of his journey. [4

   (ii) Alternatively, D may be deleted to find a lower bound. Show that this gives two possible routes. Explain why one is better than the other [6]

6. A machine is programmed to inspect all the wires in a circuit. It can only travel along wires.
   (i) List the valencies of the nodes in the circuit shown. [1]



   (ii) Hence use an appropriate algorithm to find the shortest distance the machine must travel, starting and finishing at F.
   [5]
   (iii) Write down a possible sequence of nodes that the machine can pass over.
   [2]
   (iv) Write down the number of possible pairings to consider when using this algorithm when there are (a) 6 odd nodes, (b) 10 odd nodes. Comment on your results. [3]

7. A pottery factory produces decorative mugs, bowls and plates. A mug takes 40 minutes to paint, a bowl 20 minutes and a plate 50 minutes. Each employee works a 40-hour week, and has enough room to store 80 items. No more than 30 plates should be painted by any single employee.
   The profits on each article are £3 per mug, £2 per bowl and £6 per plate.
   (i) Write down three inequalities representing the constraints on x, y and z, the numbers of mugs, bowls and plates respectively produced by one painter. [3]

   To maximise the profit, it is decided to use the Simplex algorithm.
   (ii) Write down an initial tableau, using r, s and t as slack variables. [1]
   (iii) Increasing z first, perform the Simplex process, explaining why your final tableau is optimal. [9]
   (iv) Write down the number of mugs, bowls and plates that are needed to give the maximum profit per worker, and state that profit. [3]

DECISION MATHS 1 (C) PAPER 3 : ANSWERS AND MARK SCHEME

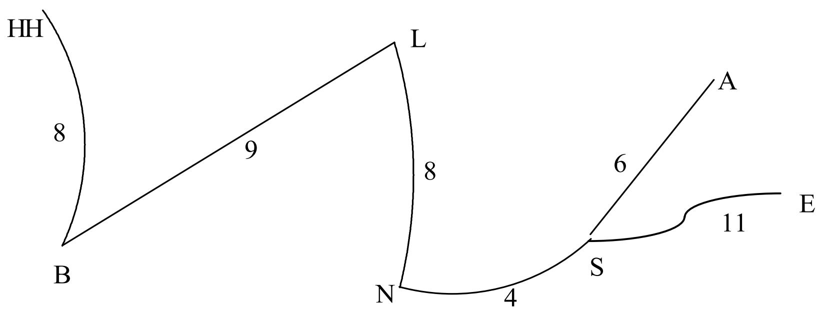
1.

Initial	B	R	L	W	H	D	M1	
1st pass	B	L	R	W	H	D	M1 A1	
2nd pass	B	H	L	R	W	D	M1	
3rd pass	B	D	H	L	R	W	A1	5

2. A: 9976 and 999706 B: 102.4 and 1.268×10^{29} B1 B1 B1 B1
A is better for sorting large sets, and B is better for small sets B1 5

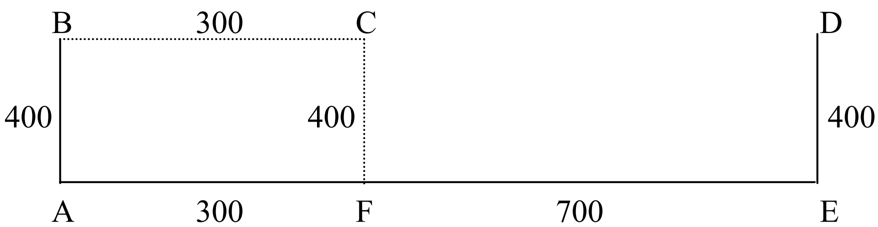
3. (i) $19x + 27y + 38z \le 2000$ B1
(ii) The cost can be written as $19X + 27y$, and thus can be treated graphically, in two dimensions, with $y \ge 20$ and $19X + 27y \le 2000$ B1
Maximum of X is 76.8; if all of X is due to parcels, this gives $z = 38$ M1 A1 A1 5

4. (i) Select arcs in order NS, SA, {NL, BH}, LB and SE M1 A1



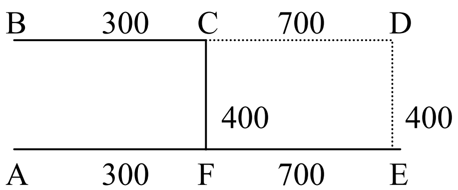
M1 A1 A1
Length = 46 A1
(ii) The SE arc is now effectively doubled in length i.e. 22; it is therefore better to use AE rather than SE (then length = 48) M1 A1 8

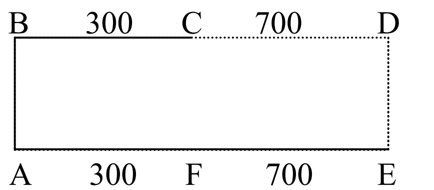
5. (i) M.S.T., with C deleted and rejoined, is M1



M1 A1
Length is 2500 m A1

(ii) There are two M.S.T.'s when D is deleted and then rejoined, as shown:





M1 A1
M1 A1

The second one is a cycle, therefore it is a solution of the TSP problem B2 10

6. (i) A2 B2 C2 D3 E4 F3 G4 H2 I2 J4
K4 L3 M3 N2 O2 P2 B1

(ii) Pairing the odd nodes : DF + LM = 4 + 5 = 9, DL + MF = 10 + 11 = 21, B1 B1
DM + FL = 15 + 6 = 21, so repeat DF and LM. Distance = 75 + 9 = 84 B1 M1 A1

(iii) e.g. F G H N M G B A E F L M L K P O J K E D C I J D E F M1 A1

(iv) (a) For 6 nodes, there are 5 x 3 = 15 possible pairing B1
(b) For 10 nodes, there are 9 x 7 x 5 x 3 = 945 possibilities B1
Number of possibilities increases rapidly with the number of nodes B1 11

7. (i) $4x + 2y + 5z \le 240$ $x + y + z \le 80$ $z \le 30$ B1 B1 B1

(ii) To maximise $P = 3x + 2y + 6z$:

P	x	y	z	r	s	t	
1	-3	-2	-6	0	0	0	0
0	4	2	5	1	0	0	240
0	1	1	1	0	1	0	80
0	0	0	1	0	0	1	30

B1

(iii) Increase z

1	-3	-2	0	0	0	6	180
0	4	2	0	1	0	-5	90
0	1	1	0	0	1	-1	50
0	0	0	1	0	0	1	30

M1 A1

Increase x

1	0	-0.5	0	0.75	0	2.25	247.5
0	1	0.5	0	0.25	0	-1.25	22.5
0	0	0.5	0	-0.25	1	0.25	27.5
0	0	0	1	0	0	1	30

M1 A1 A1

Increase y

1	1	0	0	1	0	1	270
0	2	1	0	0.5	0	-2.5	45
0	-1	0	0	-0.5	1	1.5	5
0	0	0	1	0	0	1	30

M1 A1 A1

This is a final optimal tableau, because the objective row is all positive B1

(iv) Thus max. P = £270 with no mugs, 45 bowls, 30 plates M1 A1 A1 16